On the Diffraction of Elastic Plane Pulses by a Crack of a Half Plane (Three Dimensional Problem)

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ON THE DIFFRACTION OF ELASTIC PLANE PULSES BY A CRACK OF A HALF PLANE (THREE DIMENSIONAL PROBLEM)

BY

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Abstract

The diffraction of plane elastic longitudinal (P) and transversal (S) pulses of rectangular type propagated in a three dimensional space by a crack of a half plane, was treated by using D. S. Jones's method in the diffraction of a scalar wave. The dependence of the amplitudes of the diffracted pulses on azimuth was evaluated on the electronic computer, KDC-1.

1. Introduction

In the previous paper[1], the writer has investigated the two dimensional problem of the diffraction of plane elastic P and S pulses by a crack of the half plane ; the plane of incidence of the pulses is perpendicular to the edge and the stress is equal to zero on such a half plane. In this paper, the three dimensional diffraction by the crack of the half plane which is a free surface is treated by D. S. Jones's method in the diffraction of a scalar wave[2]. That is, firstly the formal solutions for the harmonic wave are obtained by his method and, taking the inverse integral transform, the solutions are calculated for the incidence of the plane pulses of a rectangular type.

Notation :

a, b, c_R : velocities of propagation of P, S, and Rayleigh waves, respectively,

k, K : wave numbers of P and S waves, respectively,

p_{yy}, p_{yz}, p_{xy} : components of stress tensor,

u_1, u_2, u_3 : components of displacements in the x, y, z-directions, respectively,

ρ : density, and

e_x, e_y, e_z : unit vectors parallel to the x, y, z-axes, respectively.

2. Formal solution

We choose the orthogonal coordinates system, the z-axis of which coincides with the edge of the crack of the half plane, as shown in Fig. 1. On both sides of the half plane, the boundary conditions are

$$\left.\begin{array}{l} p_{yy}{}^t = p_{xy}{}^t = p_{yz}{}^t = 0 \quad \text{on} \quad y=0,\ x<0, \\ p_{yy}{}^t,\ p_{xy}{}^t,\ p_{yz}{}^t,\quad u_1{}^t,\ u_2{}^t,\ u_3{}^t \text{ are continuous on } y=0,\ x>0. \end{array}\right\} \tag{1}$$

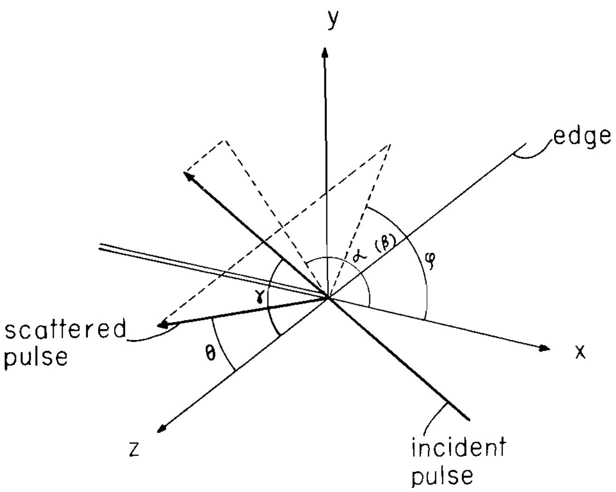


Fig. 1.

The boundary condition at the edge is such that[3]

$$p_{yy}{}^t,\ p_{xy}{}^t,\ p_{yz}{}^t \sim r^{-1/2} \qquad \text{for} \qquad r \to 0. \tag{2}$$

Now split the total potentials ϕ^t and ψ^t into the incident part and the scattered part

$$\phi^t = \phi^0 + \phi \qquad \psi^t = \psi^0 + \psi,$$

and derive the formal solutions for the diffraction problem of the P and S waves of the simple harmonic type incident from any direction to the edge of the crack of the half plane in a uniform isotropic elastic medium.

The displacement can be written in the form

$$\boldsymbol{u}^t = -\operatorname{grad}\phi^t + \operatorname{rot}\psi^t, \tag{3}$$

where the scalar potential ϕ^t and the vector potential ψ^t satisfy the equations

$$\frac{1}{a^2}\frac{\partial\phi^t}{\partial t^2}=\frac{\partial^2\phi^t}{\partial x^2}+\frac{\partial^2\phi^t}{\partial y^2}+\frac{\partial^2\phi^t}{\partial z^2},\qquad \frac{1}{b^2}\frac{\partial^2\psi^t}{\partial t^2}=\frac{\partial^2\psi^t}{\partial x^2}+\frac{\partial^2\psi^t}{\partial y^2}+\frac{\partial^2\psi^t}{\partial z^2},\tag{4}$$

Then, consider an auxiliary coordinates system x, y, t_1 which is advancing in the direction of the negative z-axis and is connected with the above by means of the following formula.

$$x=x$$
$$y=y$$
$$t_1=t-\frac{z}{c}.$$

From the physical point of view, the state is static in such a moving system ; that is, the displacement depends only on the three variables, x, y, t_1[4]. Inserting this result into the wave equations (4), we obtain for the potentials (or displacements)

$$\left.\begin{aligned}&\frac{1}{a_1^2}\frac{\partial^2\phi^t}{\partial t_1^2}=\frac{\partial^2\phi^t}{\partial x^2}+\frac{\partial^2\phi^t}{\partial y^2}, &&\frac{1}{b_1^2}\frac{\partial^2\psi^t}{\partial t_1^2}=\frac{\partial^2\psi^t}{\partial x^2}+\frac{\partial\psi^t}{\partial y^2},\\ &\frac{1}{a_1^2}=\frac{1}{a^2}-\frac{1}{c^2}, &&\frac{1}{b_1^2}=\frac{1}{b^2}-\frac{1}{c^2},\end{aligned}\right\}\tag{5}$$

The potentials ϕ, ψ as the superposition of plane waves can be written in the form

$$\left.\begin{aligned}\phi&=\int_{-\infty}^{\infty}J_{1,2}(\lambda,k)e^{-ia_1kt_1+i(\lambda x+\sqrt{k^2-\lambda^2}|y|)}dkd\lambda\\ \psi_1&=\int_{-\infty}^{\infty}L_{1,2}(\lambda,K)e^{-ib_1Kt_1+i(\lambda x+\sqrt{K^2-\lambda^2}|y|)}dKd\lambda\\ \psi_2&=\int_{-\infty}^{\infty}M_{1,2}(\lambda,K)e^{-ib_1Kt_1+i(\lambda x+\sqrt{K^2-\lambda^2}|y|)}dKd\lambda\\ \psi_3&=\int_{-\infty}^{\infty}N_{1,2}(\lambda,K)e^{-ib_1Kt_1+i(\lambda x+\sqrt{K^2-\lambda^2}|y|)}dKd\lambda\end{aligned}\right\}\tag{6}$$

$$a_1k=b_1K$$

where 1 for $y>0$, 2 for $y<0$. In order to satisfy the radiation condition at $y\to\infty$, we must interprete the integrals in (6) as the limits of those along the path L in the complex plane shown in Fig. 2, when semicircles around seven singular points $\pm k$, $\pm K$, $\pm\lambda_R$, κ are made vanishingly small. Thus the stresses

$$\left.\begin{aligned}
p_{yy}&=\int_{-\infty}^{\infty}P_{yy}e^{i\lambda x}d\lambda=2\rho b^2\int_{-\infty}^{\infty}\Big\{-\Big(\frac{a_1^2k^2}{c^2}-\frac{a_1^2k^2}{2b^2}+\lambda^2\Big)J_{1,2}\mp\frac{a_1k}{c}\times\\
&\qquad\sqrt{K_2-\lambda^2}L_{1,2}\pm\lambda\sqrt{K^2-\lambda^2}N_{1,2}\Big\}e^{i\lambda x}d\lambda\\
p_{xy}&=\int_{-\infty}^{\infty}P_{xy}e^{i\lambda x}d\lambda=2\rho b^2\int_{-\infty}^{\infty}\Big\{\pm2\lambda\sqrt{k^2-\lambda^2}J_{1,2}-(K^2-2\lambda^2)N_{1,2}\\
&\qquad-\frac{a_1k}{c}\lambda L_{1,2}\pm\frac{a_1k}{c}\sqrt{K^2-\lambda^2}M_{1,2}\Big\}e^{i\lambda x}d\lambda\\
p_{yz}&=\int_{-\infty}^{\infty}P_{yz}e^{i\lambda x}d\lambda=2\rho b^2\int_{-\infty}^{\infty}\Big\{\pm2\frac{a_1k}{c}\sqrt{k^2-\lambda^2}J_{1,2}-\Big(\frac{a_1^2k^2}{c^2}\\
&\qquad-K^2+\lambda^2\Big)L_{1,2}\mp\lambda\sqrt{K^2-\lambda^2}M_{1,2}+\frac{a_1k}{c^2}\lambda N_{1,2}\Big\}e^{i\lambda x}d\lambda,
\end{aligned}\right\}\quad(7)$$

$$J_{1,2}=J^1\pm J^2,\qquad M_{1,2}=M^1\pm M^2$$
$$L_{1,2}=\pm L^1+L^2,\qquad N_{1,2}=\pm N^1+N^2.$$

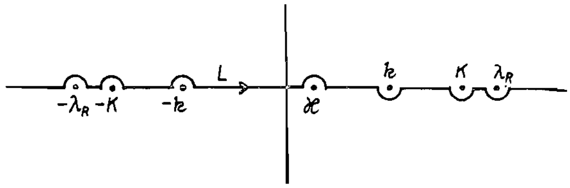


Fig. 2.

Insert (7) into the boundary conditions, we obtain

$$\left.\begin{aligned}
&-\Big\{\frac{a_1^2k^2}{c^2}-\frac{a_1^2k^2}{2b^2}+\lambda^2\Big\}J^2+\sqrt{K^2-\lambda^2}\Big\{\lambda N^2-\frac{a_1k}{c}L^2\Big\}=0\\
&2\lambda\sqrt{k^2-\lambda^2}J^1-\sqrt{K^2-\lambda^2}\Big\{\sqrt{K^2-\lambda^2}N^1-\frac{a_1k}{c}M^1\Big\}+\lambda\Big\{\lambda N^1-\frac{a_1k}{c}L^1\Big\}=0\\
&\frac{2a_1k}{c}\sqrt{k^2-\lambda^2}J^1-\sqrt{K^2-\lambda^2}\{\lambda M^1-\sqrt{K^2-\lambda^2}L^1\}+\frac{a_1k}{c}\Big\{\lambda N^1-\frac{a_1k}{c}L^1\Big\}=0.
\end{aligned}\right\}\quad(9)$$

Then, J, L, M, N, can be expressed in terms of three unknown functions. That is,

$$\left.\begin{aligned}
&J^1=\Big\{-\frac{a_1^2k^2}{2c^2}+\frac{K^2}{2}-\lambda^2\Big\}R_2,\\
&J^2=-\lambda\sqrt{K^2-\lambda^2}R_1\\
&\lambda M^1-\sqrt{K^2-\lambda^2}L^1=\frac{a_1k}{c}\sqrt{k^2-\lambda^2}\sqrt{K^2-\lambda^2}R_2\\
&\lambda M^2-\sqrt{K^2-\lambda^2}L^2=\lambda R_3\\
&\lambda N^1-\frac{a_1k}{c}L^1=\sqrt{k^2-\lambda^2}\Big(\frac{a_1k^2}{c^2}+\lambda^2\Big)R_2
\end{aligned}\right\}\quad(10)$$

$$\left.\begin{aligned}
&\lambda N^2-\frac{a_1k}{c}L^2=-\lambda\left(\frac{a_1{}^2k^2}{2c^2}-\frac{K^2}{2}+\lambda^2\right)R_1\\
&\sqrt{K^2-\lambda^2}N^1-\frac{a_1k}{c}M^1=\lambda\sqrt{k^2-\lambda^2}\sqrt{K^2-\lambda^2}R_2\\
&\sqrt{K^2-\lambda^2}N^2-\frac{a_1k}{c}M^2=-\sqrt{K^2-\lambda^2}\left\{\frac{a_1{}^2k^2}{2c^2}-\frac{K^2}{2}+\lambda^2\right\}R_1-\frac{a_1k}{c}R_3.
\end{aligned}\right\}$$

Next, split the transformed stresses into the following two parts

$$\left.\begin{aligned}
&P_{yy}(\lambda,y)=P_{yy}{}^{+}(\lambda,y)+P_{yy}{}^{-}(\lambda,y),\\
&P_{yy}{}^{+}(\lambda,y)=\frac{1}{2\pi}\int_{-\infty}^{0}p_{yy}e^{-i\lambda x}dx,\\
&P_{yy}{}^{-}(\lambda,y)=\frac{1}{2\pi}\int_{0}^{\infty}p_{yy}e^{-i\lambda x}dx,\\
&\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}\right\}\tag{10}$$

For brevity, we shall sometimes write $f(\lambda)$ or $f(y)$ instead of $f(\lambda, y)$ when there is danger of confusion. An expression like $f(\pm 0)$ will always refer to the value of $f(\lambda, y)$ for $y=0$, where $+0$ means the limit as y tends to zero approached from positive values of y, etc.. Now we define the λ-plane, not lower than the real axis as limits of the line L shown in Fig. 2, to be the upper one, and the plane not above the real axis to be the lower one. When the solutions satisfy the radiation condition, $P_{yy}{}^{+}$ is regular in the upper plane, $P_{yy}{}^{-}$ regular in the lower. We also split the displacements into the following two parts in the same way

$$\left.\begin{aligned}
&U(\lambda)=U^{+}(\lambda)+U^{-}(\lambda),\\
&U^{+}(\lambda)=\frac{1}{2\pi}\int_{-\infty}^{0}ue^{-i\lambda x}dx,\\
&U^{-}(\lambda)=\frac{1}{2\pi}\int_{0}^{\infty}ue^{-i\lambda x}dx,
\end{aligned}\right\}\tag{11}$$

On applying the above definitions to (10), (11) and considering the boundary conditions, we find

$$\begin{aligned}
P_{yy}{}^{+}(0)+P_{yy}{}^{-}(0)=2\rho b^2&\left[\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)^2\right.\\
&\left.+\left(\frac{a_1{}^2k^2}{c^2}+\lambda^2\right)\sqrt{k^2-\lambda^2}\sqrt{K^2-\lambda^2}\right]R_2
\end{aligned}\tag{12$_1$}$$

$$\begin{aligned}
P_{xy}{}^{+}(0)+P_{xy}{}^{-}(0)=-2\rho b^2&\left[\left(\frac{K^2}{2}-\lambda^2\right)\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)\right.\\
&\left.+\lambda^2\sqrt{k^2-\lambda^2}\sqrt{K^2-\lambda^2}\right]R_1+\rho b^2\frac{a_1k}{c}\sqrt{K^2-\lambda^2}R_3
\end{aligned}\tag{12$_2$}$$

$$P_{yz}{}^{+}(0)+P_{yz}{}^{-}(0)=\rho b\frac{a_1k}{c}\lambda\left[\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right.$$

$$-2\sqrt{k^2-\lambda^2}\sqrt{K^2-\lambda^2}\,R_1-\rho b^2\lambda\sqrt{K^2-\lambda^2}R_3, \tag{12}_3$$

$$U^+(+0)-U^+(-0)=-2i\left[\sqrt{K^2-\lambda^2}\left(\frac{a_1{}^2k^2}{2c^2}-\frac{K^2}{2}\right)R_1+\frac{a_1k}{c}R_3\right] \tag{13}_1$$

$$V^+(+0)-V^+(-0)=-i\left(\frac{a_1{}^2k^2}{c^2}+K^2\right)\sqrt{k^2-\lambda^2}R^2 \tag{13}_2$$

$$W^+(+0)-W^+(-0)=2i\lambda\left[\frac{a_1k}{c}\sqrt{K^2-\lambda^2}R_1+R_3.\right]. \tag{13}_3$$

For simplicity, introduce the notation

$$\left.\begin{aligned} U^+(+0)-U^+(-0)&=D_1{}^+\\ V^+(+0)-V^+(-0)&=D_2{}^+\\ W^+(+0)-W^+(-0)&=D_3{}^+.\end{aligned}\right\} \tag{14}$$

Eliminate R_2 between (12) and (13). Then

$$P_{yy}{}^+(0)+P_{yy}{}^-(0)=i\frac{\rho b^2(K^2-k^2)\sqrt{K^2-\lambda^2}F(\lambda)D_2{}^+}{\left(\dfrac{a_1{}^2k^2}{c^2}+K^2\right)} \tag{15}$$

$$F(\lambda)=\frac{2G(\lambda)}{(K^2-k^2)\sqrt{(k^2-\lambda^2)(K^2-\lambda^2)}}=F^+(\lambda)F^-(\lambda), \tag{16}$$

where

$$G(\lambda)=\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)^2+\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)\sqrt{(k^2-\lambda^2)(K^2-\lambda^2)}. \tag{17}$$

First, we investigate the diffraction resulting from the incidence of the plane longitudinal wave in the following form

$$\left.\begin{aligned}\phi^0&=e^{-ia_1kt_1+i(\kappa x+\sqrt{k^2-\kappa^2}y)}\\ \psi^0&=0,\end{aligned}\right\} \tag{18}$$

and look for the function R_2. In the equation (14), $P_{yy}{}^+(0)$ is known from the boundary condition (1). In fact

$$P_{yy}{}^+(0)=\frac{1}{2\pi}\int_{-\infty}^{0}(-p_{yy}{}^0)e^{-i\lambda x}dx=\frac{\rho b^2}{\pi}\int_{-\infty}^{0}A_1e^{i(\kappa-\lambda)x}dx=\frac{\rho b^2A_1}{\pi i(\kappa-\lambda)} \tag{19}$$

where

$$A_1=-\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\kappa^2\right).$$

The equation (14) now becomes

$$\left.\frac{P_{yy}{}^-(0)}{\sqrt{K-\lambda}F^-(\lambda)}+g_1(\lambda)=\frac{i\rho b^2(K^2-k^2)\sqrt{K+\lambda}D_2{}^+F^+(\lambda)}{\left(K^2+\dfrac{a_1{}^2k^2}{c^2}\right)},\right\} \tag{20}$$

where

$$g_1(\lambda)=\frac{\rho b^2 A_1}{\pi i(\kappa-\lambda)\sqrt{K-\lambda}\,F^-(\lambda)}.$$

Next consider the decomposition of the function $F(\lambda)$ in the form of a product $F^+(\lambda)\ F^-(\lambda)$ by means of the splitting of $f(\lambda)=\log F(\lambda)$ into the form of a sum, where $F^+(\lambda)$ and $F^-(\lambda)$ are regular in the upper and lower half planes, respectively. The singular points of $\log F(\lambda)$ are $\pm k$, $\pm K$, $\pm\lambda_R$, zeros of $F(\lambda)$, and

$$F(\lambda)\sim 1+\frac{\text{const.}}{\lambda^2} \qquad \text{for} \quad \lambda\to\infty.$$

$f(\lambda)$ can be written in the form by Cauchy's theorem

$$\begin{aligned} f(\lambda)=\log F(\lambda)&=\frac{1}{2\pi i}\int_{\Gamma^+}\frac{\log F(z)}{z-\lambda}dz-\frac{1}{2\pi i}\int_{\Gamma^-}\frac{\log F(z)}{z-\lambda}dz \\ &=f^+(\lambda)+f^-(\lambda), \end{aligned} \tag{21}$$

where contour Γ^+, Γ^- is shown in Fig. 3, λ is contained in the domain enclosed by Γ^+ and Γ^-. Taking the singular points $\pm k$, $\pm K$, $\pm\lambda_R$ into consideration, we can write (21) in the following way

$$\begin{aligned} f^{+,-}(\lambda)&=\pm\frac{1}{2\pi i}\int_{\Gamma^{+,-}}\log\left\{1+\frac{\left(\dfrac{K^2}{2}-\dfrac{a_1{}^2k^2}{2c^2}-z^2\right)^2}{\left(\dfrac{a_1{}^2k^2}{c^2}+z^2\right)\sqrt{(k^2-z^2)(K^2-z^2)}}\right\}\frac{dz}{z-\lambda} \\ &=\log\frac{\lambda_R\pm\lambda}{K\pm\lambda}+\frac{1}{\pi}\int_{\mp k}^{\mp K}\tan^{-1}\left\{\frac{\left(\dfrac{K^2}{2}-\dfrac{a_1{}^2k^2}{2c^2}-z^2\right)^2}{\left(\dfrac{a_1{}^2k^2}{c^2}+z^2\right)\sqrt{(k^2-z^2)(K^2-z^2)}}\right\}\frac{dz}{z-\lambda} \end{aligned} \tag{22}$$

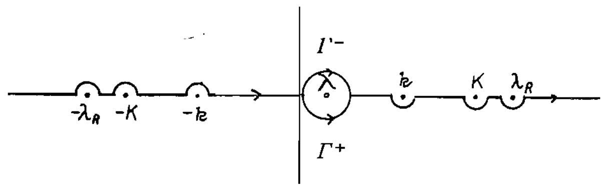


Fig. 3.

Split $g_1(\lambda)$ into two parts in the same way as (21)

$$\begin{aligned} &g_1(\lambda)=g_1{}^+(\lambda)+g_1{}^-(\lambda) \\ &\quad g_1{}^+(\lambda)=\frac{\rho b^2A_1}{\pi i\sqrt{K-\kappa}\,(\kappa-\lambda)F^-(\kappa)} \\ &\quad g_1{}^-(\lambda)=\frac{\rho b^2A_1}{\pi i(\kappa-\lambda)}\left\{\frac{1}{\sqrt{K-\lambda}\,F^-(\lambda)}-\frac{1}{\sqrt{K-\kappa}\,F^-(\kappa)}\right\} \end{aligned} \tag{23}$$

where $g_1^+(\lambda)$ is regular in the upper, $g_1^-(\lambda)$ is regular in the lower. Insert (23) in (19) and rearange

$$I_1(\lambda)=\frac{P_{yy}^-(0)}{\sqrt{K-\lambda}F^-(\lambda)}+g_1^-(\lambda)=\frac{i\rho b^2(K^2-k^2)\sqrt{K+\lambda}D_2^+F^+(\lambda)}{\left(K^2+\frac{a_1^2k^2}{c^2}\right)}-g_1^+(\lambda). \quad (24)$$

In this form the function $I_1(\lambda)$ is regular in the upper plane and also regular in the lower, i.e., in the whole of the plane, since these two half planes overlap. And we proceed to examine the behavior of the function $I_1(\lambda)$ as λ tends to infinity.

From the edge condition and the Abelian theorem[5]

$$I_1(\lambda)\to\frac{1}{|\lambda|} \qquad \text{for} \quad \lambda\to\infty \quad (25)$$

$I_1(\lambda)$ tends to zero as λ tends to infinity in any direction. Hence, from Liouville's theorem $I_1(\lambda)$ must be identically zero, i.e.

$$R_2=\frac{-A_1\sqrt{K-\lambda}F^-(\lambda)}{2\pi i(\lambda-\kappa)\sqrt{K-\kappa}\,F^-(\kappa)G(\lambda)} \quad (26)$$

Now we seek for R_1, R_3. Add the $(12)_2$ and $(12)_3$ multiplied by $\frac{a_1k}{c\lambda}$ and subtract $(12)_3$ multiplied by $\frac{c\lambda}{a_1k}$ from $(12)_2$ to find

$$P_{xy}^+(0)+P_{xy}^-(0)+\frac{a_1k}{c\lambda}\{P_{yz}^+(0)+P_{yz}^-(0)\}=-2\rho b^2G(\lambda)R_1 \quad (27)_1$$

$$P_{xy}^+(0)+P_{xy}^-(0)-\frac{c\lambda}{a_1k}\{P_{yz}^+(0)+P_{yz}^-(0)\}=-\rho b^2(K^2-\lambda^2)$$
$$\times\left(\frac{K^2}{2}-\frac{a_1^2k^2}{2c^2}-\lambda^2\right)R_1+\rho b^2\sqrt{K^2-\lambda^2}\left(\frac{a_1k}{c}+\frac{c\lambda^2}{a_1k}\right)R_3. \quad (27)_2$$

Eliminate R_1, R_3 between (13) and $(27)_1$, and rearange the resulting equation. Then,

$$I_2(\lambda)=\frac{\lambda P_{xy}^-(0)+\frac{a_1k}{c}P_{yz}^-(0)}{\sqrt{k-\lambda}F^-(\lambda)}+g_2^-(\lambda)$$
$$=i\rho b^2K^{-2}(K^2-k^2)\lambda\sqrt{k+\lambda}\left(D_1^++\frac{a_1k}{c\lambda}D_3^+\right)F^+(\lambda)-g_2^+(\lambda)$$

$$g_2^+(\lambda)=\frac{\rho b^2}{\pi i}\,\frac{\lambda A_2+\frac{a_1k}{c}A_3}{\sqrt{k-\kappa}\,(\kappa-\lambda)F^-(\kappa)},$$

$$g_2^-(\lambda)=\frac{\rho b^2}{\pi i}\left(\lambda A_2+\frac{a_1k}{c}A_3\right)\left\{\frac{1}{(\kappa-\lambda)\sqrt{k-\lambda}F^-(\lambda)}-\frac{1}{(\kappa-\lambda)\sqrt{k-\kappa}\,F^-(\kappa)}\right\}. \quad (28)$$

By the same procedure, we rearange $(27)_2$. This gives

$$I_3(\lambda)=\frac{P_{xy}^-(0)-\dfrac{c\lambda}{a_1k}P_{yz}^-(0)}{\sqrt{K-\lambda}}+g_3^-(\lambda)=\frac{i\rho b^2}{2}\sqrt{K+\lambda}\left\{D_1^+-\frac{c\lambda}{a_1k}D_3^+\right\}-g_3^+(\lambda),$$

$$g_3^+(\lambda)=\frac{\rho b^2\left(A_2-\dfrac{c\lambda}{a_1k}A_3\right)}{\pi i\sqrt{K-\kappa}\ (\kappa-\lambda)},\quad g_3^-(\lambda)=\frac{\rho b^2\left(A_2-\dfrac{c\lambda}{a_1k}A_3\right)}{\pi i(\kappa-\lambda)}\left\{\frac{1}{\sqrt{K-\lambda}}-\frac{1}{\sqrt{K-\kappa}}\right\}. \tag{29}$$

From the edge condition

$$\left.\begin{array}{l} D_1^+\sim\lambda^{-3/2} \\ D_3^+\sim\lambda^{-3/2} \end{array}\right. \quad \text{for} \quad |\lambda|\to\infty. \tag{30}$$

In this from $I_2(\lambda)$, $I_3(\lambda)$ are regular. Hence, from Liouville's theorem $I_2(\lambda)$, $I_3(\lambda)$ must be constants, i.e.

$$\left.\begin{aligned} I_2(\lambda)&=\frac{\rho b^2\dfrac{a_1k}{c}B_1}{\pi i\sqrt{k-\kappa}\ F^-(\kappa)} \\ I_3(\lambda)&=\frac{\rho b^2B_2}{\pi i\sqrt{K-\kappa}}. \end{aligned}\right\} \tag{31}$$

Then,

$$\begin{aligned}\frac{iD_1^+}{2}=&-\frac{\lambda\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}\left(K^2+\dfrac{a_1^2k^2}{c^2}\right)\left\{\lambda A_2+\dfrac{a_1k}{c}A_3-\dfrac{a_1k}{c}(\lambda-\kappa)B_1\right\}F^-(\lambda)}{4\pi i\sqrt{k-\kappa}\,(\lambda-\kappa)\left(\lambda^2+\dfrac{a_1^2k^2}{c^2}\right)F^-(\kappa)G(\lambda)} \\ &-\frac{\dfrac{a_1^2k^2}{c^2}\left(A_2-\dfrac{c\lambda}{a_1k}A_3-(\lambda-\kappa)B_2\right)}{\pi i\sqrt{K-\kappa}\ \sqrt{K+\lambda}\,(\lambda-\kappa)\left(\lambda^2+\dfrac{a_1^2k^2}{c^2}\right)}.\end{aligned} \tag{32}$$

Determine the unknown constants, B_1, B_2 so that the right hand side of (32) may satisfy the condition of regularity in the upper plane, i.e. at the point $\lambda=\dfrac{a_1k}{c}i$. Therefore, we must put the expression (32) equal to zero for $\lambda=\dfrac{a_1k}{c}i$. Then, putting both the real and imaginary parts of (32) equal to zero, we find

$$\left.\begin{aligned} B_1&=\sigma_1-\frac{\sigma_1+\sigma_2\tau_2}{\tau_1} \\ B_2&=\sigma_2-(\sigma_1\tau_2+\sigma_2\tau_1)+B_1\tau_2, \end{aligned}\right\} \tag{33}$$

where

$$\sigma_1 = -\frac{\frac{a_1 k}{c} A_2 + \kappa A_3}{\kappa^2 + \frac{a_1^2 k^2}{c^2}} \qquad \sigma_2 = \frac{\frac{a_1 k}{c} A_3 - \kappa A_2}{\kappa^2 + \frac{a_1^2 k^2}{c^2}}$$

$$\tau_1 = \frac{K\sqrt{K-\kappa}}{\sqrt{2(k-\kappa)(K^2-k^2)}} F^-(\kappa), \qquad \tau_2 = \frac{\frac{a_1 k}{c}\sqrt{K-\kappa}}{\sqrt{2(k-\kappa)(K^2-k^2)}\, F^-(\kappa)}. \tag{34}$$

Thus, we obtain R_1, R_3 from (13), (14), (28), (29), (31).

$$R_1 = \frac{\sqrt{k-\lambda}\left\{\lambda A_2 + \frac{a_1 k}{c} A_3 - \frac{a_1 k}{c}(\lambda-\kappa) B_1\right\} F^-(\lambda)}{2\pi i \sqrt{k-\kappa}\ \lambda(\lambda-\kappa) F^-(\kappa) G(\lambda)} \tag{35}$$

$$R_3 = \frac{\frac{a_1 k}{c}}{\left(\lambda^2 + \frac{a_1^2 k^2}{c^2}\right)} \left\{ \frac{-\left(A_2 - \frac{c\lambda}{a_1 k} A_3\right) + (\lambda-\kappa) B_2}{\pi i (\lambda-\kappa)\sqrt{K-\kappa}\sqrt{K-\lambda}} + \sqrt{K^2-\lambda^2}\left(\frac{K^2}{2} - \frac{a_1^2 k^2}{2c^2} - \lambda^2\right) R_1 \right\} \tag{36}$$

Finally, the scattered part for the harmonic P plane wave is

$$\begin{aligned}
\boldsymbol{u}^p &= -\operatorname{grad}\int_{-}^{\infty} \frac{1}{2\pi i}\sqrt{\frac{K-\lambda}{K-\kappa}}\, \frac{F^-(\lambda)}{F^-(\kappa)}\, \frac{H_1(\lambda)}{(\lambda-\kappa)\cdot G(\lambda)} \\
&\quad \times e^{-i a_1 k t_1 + i(\lambda x + \sqrt{k^2-\lambda^2}|y|)} d\lambda \\
u_1{}^s &= \int_{-\infty}^{\infty} \pm \frac{\frac{a_1^2 k^2}{c^2}(\sqrt{k^2-\kappa^2} - B^2)}{\pi\sqrt{K-\kappa}\sqrt{K+\lambda}\left(\lambda^2 + \frac{a_1^2 k^2}{c^2}\right)} \\
&\quad + \left.\frac{\lambda\sqrt{k-\lambda}\sqrt{K^2-\lambda^2} F^-(\lambda) H_2(\lambda)}{2\pi\sqrt{K-\kappa}\, F^-(\kappa)(\lambda-\kappa) G(\lambda)\left(\lambda^2 + \frac{a_1^2 k^2}{c^2}\right)}\right] \\
&\quad \times e^{-i b_1 K t_1 + i(\lambda x + \sqrt{K^2-\lambda^2}|y|)} d\lambda \\
u_2{}^s &= \int_{-\infty}^{\infty} \frac{\mp}{2\pi}\sqrt{\frac{k-\lambda}{K-\kappa}}\, \frac{F^-(\lambda)}{F^-(\kappa)}\, \frac{H_2(\lambda)}{(\lambda-\kappa) G(\lambda)} \\
&\quad e^{-i b_1 K t_1 + i(\lambda x + \sqrt{K^2-\lambda^2}|y|)} d\lambda \\
u_3{}^s &= \int_{-\infty}^{\infty} \left[\mp \frac{\frac{a_1 k \lambda}{c}\{\sqrt{k^2-\kappa^2} - B_2\}}{\pi\sqrt{K-\kappa}\sqrt{K+\lambda}\left(\lambda^2 + \frac{a_1^2 k^2}{c^2}\right)} \right. \\
&\quad + \left.\frac{\frac{a_1 k}{c}\sqrt{k-\lambda}\sqrt{K^2-\lambda^2} F^-(\lambda) H_2(\lambda)}{2\pi\sqrt{K-\kappa}\, F^-(\kappa)\left(\lambda^2 + \frac{a_1^2 k^2}{c^2}\right)(\lambda-\kappa) G(\lambda)}\right] \\
&\quad \times e^{-i b_1 K t_1 + i(\lambda x + \sqrt{K^2-\lambda^2}|y|)} d\lambda
\end{aligned} \tag{37}$$

$$\left.\begin{aligned}
H_1(\lambda)&=\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\kappa^2\right)\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)\pm\kappa\sqrt{(k+\kappa)(K-\kappa)}\\
&\times\left(\lambda+\frac{a_1{}^2k^2}{c^2\kappa}\right)\sqrt{(k-\lambda)(K+\lambda)}\pm\frac{B_1\dfrac{a_1k}{c}\sqrt{K-\kappa}\sqrt{(k-\kappa)(K+\lambda)}(\lambda-\kappa)}{\sqrt{k-\kappa}}\\
H_2(\lambda)&=\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\kappa^2\right)\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)\sqrt{(k+\lambda)(K-\lambda)}\\
&\quad\mp\sqrt{(k+\kappa)(K-\kappa)}\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)\\
&\quad\times\left\{\lambda\kappa+\frac{a_1{}^2k^2}{c^2}+\frac{B_1\dfrac{a_1k}{c}(\lambda-\kappa)}{\sqrt{k^2-\kappa^2}}\right\}
\end{aligned}\right\}\tag{38}$$

Specifying the incident S wave in the form

$$\left.\begin{aligned}
\psi_3&=e^{-ib_1Kt_1+i(\kappa x+\sqrt{K^2-\kappa^2}|y|)}\\
\phi&=\psi_1=\psi_2=0,
\end{aligned}\right\}\tag{39}$$

we find also in a similar way

$$\left.\begin{aligned}
u^P&=-\operatorname{grad}\int_{-\infty}^{\infty}\frac{1}{2\pi i}\sqrt{\frac{K-\lambda}{k-\kappa}}\frac{F^-(\lambda)K_1(\lambda)}{F^-(\kappa)(\lambda-\kappa)G(\lambda)}\\
&\quad\times e^{-ia_1kt_1+i(\lambda x+\sqrt{k^2-\lambda^2}|y|)}d\lambda\\
u_1{}^s&=\int_{-\infty}^{\infty}\left[\pm\frac{\dfrac{a_1{}^2k^2}{c^2}\left\{\dfrac{K^2}{2}-\kappa^2+\dfrac{\kappa\lambda}{2}-B_2(\lambda-\kappa)\right\}}{\pi\sqrt{K-\kappa}\sqrt{K+\lambda}(\lambda-\kappa)\left(\lambda^2+\dfrac{a_1{}^2k^2}{c^2}\right)}\right.\\
&\quad\left.+\frac{\kappa\lambda\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}F^-(\lambda)K_2(\lambda)}{2\pi\sqrt{k-\kappa}F^-(\kappa)(\lambda-\kappa)\left(\lambda^2+\dfrac{a_1{}^2k^2}{c^2}\right)G(\lambda)}\right]\\
&\quad\times e^{-ib_1Kt_1+i(\lambda x+\sqrt{K^2-\lambda^2}|y|)}d\lambda\\
u_2{}^s&=\int_{-\infty}^{\infty}\frac{\mp}{2\pi}\sqrt{\frac{k-\lambda}{k-\kappa}}\frac{\kappa F^-(\lambda)K_2(\lambda)}{F^-(\kappa)(\lambda-\kappa)G(\lambda)}\\
&\quad\times e^{-ib_1Kt_1+i(\lambda x+\sqrt{K^2-\lambda^2}|y|)}d\lambda\\
u_3{}^s&=\int_{-}^{\infty}\left[\mp\frac{\dfrac{a_1k}{c}\lambda\left\{\dfrac{K^2}{2}-\kappa^2+\dfrac{\kappa\lambda}{2}-B_2(\lambda-\kappa)\right\}}{\pi\sqrt{K-\kappa}\sqrt{K+\lambda}(\lambda-\kappa)\left(\lambda^2+\dfrac{a_1{}^2k^2}{c^2}\right)}\right.\\
&\quad\left.+\frac{\dfrac{a_1k\kappa}{c}\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}F^-(\lambda)K_2(\lambda)}{2\pi\sqrt{k-\kappa}F^-(\kappa)(\lambda-\kappa)\left(\lambda^2+\dfrac{a_1{}^2k^2}{c^2}\right)G(\lambda)}\right]\\
&\quad\times e^{-ib_1Kt_1+i(\lambda x+\sqrt{K^2-\lambda^2}|y|)}d\lambda,
\end{aligned}\right\}\tag{40}$$

$$
\left.\begin{aligned}
K_1(\lambda) &= \kappa\sqrt{(k-\kappa)(K+\kappa)}\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)\\
&\quad\mp\left\{\left(\frac{K^2}{2}-\kappa^2\right)\frac{\lambda}{\kappa}-\frac{a_1{}^2k^2}{2c^2}\right\}\lambda\sqrt{(k-\lambda)(K+\lambda)}\\
&\quad\pm B_1\frac{a_1k}{c}\sqrt{(k-\lambda)(K+\lambda)}\,(\lambda-\kappa)\\
K_2(\lambda) &= \sqrt{(k-\kappa)(K+\kappa)}\ \sqrt{(k+\lambda)(K-\lambda)}\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)\\
&\quad\pm\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)\times\left\{\left(\frac{K^2}{2}-\kappa^2\right)\frac{\lambda}{\kappa}-\frac{a_1{}^2k^2}{2c^2}-\frac{B_1a_1k(\lambda-\kappa)}{\kappa c}\right\},
\end{aligned}\right\}\tag{41}
$$

$$
A_1=-\kappa\sqrt{K^2-\kappa^2}\qquad A_2=\frac{K^2}{2}-\kappa^2\qquad A_3=-\frac{a_1k\kappa}{2c}.
$$

3. Transformation of the integrals

In this section we derive the solutions of the diffraction of the P and S pulses from the known solutions of the harmonic waves by the inverse integral transform. Assume that the incident P plane pulse has the form

$$
\left.\begin{aligned}
\boldsymbol{u}^0 &= \boldsymbol{e}D\left\{t-\frac{1}{a}(\cos\alpha'\cdot x+\cos\beta'\cdot y+\cos\gamma'\cdot z)\right\}\\
&= \frac{\boldsymbol{e}}{\pi}\int_{-\infty}^{\infty}\frac{\sin(\varepsilon a_1k)}{k}e^{-ia_1k\left(t-\frac{ex}{a}\right)}dk
\end{aligned}\right\}\tag{42}
$$

where

$$
\begin{aligned}
&\boldsymbol{e}=\cos\alpha'\cdot\boldsymbol{e}_x+\cos\beta'\boldsymbol{e}_y+\cos\gamma'\boldsymbol{e}_z,\\
&\quad\cos\alpha'=\cos\alpha\cdot\sin\gamma,\qquad \cos\beta'=\sin\alpha\cdot\sin\gamma,\qquad \cos\gamma'=\cos\gamma\\
&\quad x=r\sin\theta\cos\varphi,\qquad y=r\sin\theta\sin\varphi,\qquad z=r\cos\theta.
\end{aligned}
$$

The scattered pulse for the incidence of the pulse of the form (42) can be obtained by the inverse integral transform of the solution (37) for the incidence of the harmonic wave. Namely, the P part is for $\varphi>0$

$$
\boldsymbol{u}^P=\mathrm{grad}\frac{1}{\pi i}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{P(\lambda,\kappa)}{k(\lambda-\kappa)}\frac{\sin\varepsilon a_1k}{k}e^{-ia_1kt+i\left(\lambda x+\sqrt{k^2-\lambda^2}|y|+\frac{a_1k}{c}z\right)}\times dk\cdot d\lambda
$$

$$
P(\lambda,\kappa)=\frac{1}{2\pi i}\sqrt{\frac{K-\lambda}{K-\kappa}}\frac{F^-(\lambda)H_1(\lambda,\kappa)}{F^-(\kappa)G(\lambda)}.\tag{43}
$$

For $\varphi<0$, $-\varphi$ takes the places of φ.

Exchange the order of integration, change the variable λ to δ defined by $\lambda=k\cos\delta=k\cos(\varphi+is)$ and deform the contour C to the line $\varphi=$const., $-\infty<s<\infty$ as in Fig. 4. Integrate with respect to k, to find

$$u^P = -e\frac{H(\alpha)}{G(\alpha)}D\left(t-\frac{e\cdot x}{a}\right) - \mathrm{grad}\int_{s_1}^{s_2}\frac{P(\delta,\alpha)\sin(\varphi+is)}{k(\cos(\varphi+is)-\cos\alpha)}ds$$

$$-\mathrm{grad}\int_{-s_2}^{-s_1}\frac{P(\delta_1\alpha)\sin(\varphi+is)}{k(\cos(\varphi+is)-\cos\alpha)}ds. \tag{44}$$

$$s_1=\cosh^{-1}\frac{a(t-\varepsilon)-r\cos\gamma\cos\theta}{r\sin\gamma\sin\theta},\qquad s_2=\cosh^{-1}\frac{a(t+\varepsilon)-r\cos\gamma\cos\theta}{r\sin\gamma\sin\theta}$$

$$\kappa=k\cos\alpha=K\cos\beta,$$

where the first term expresses the reflected pulse and vanishes for $|\varphi|<\alpha$, and the second and the third express the diffracted pulse and are the complex conjugate of each other.

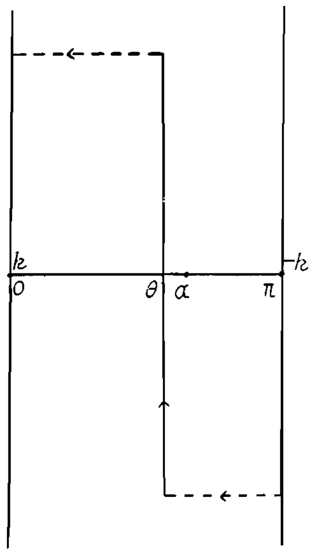


Fig. 4.

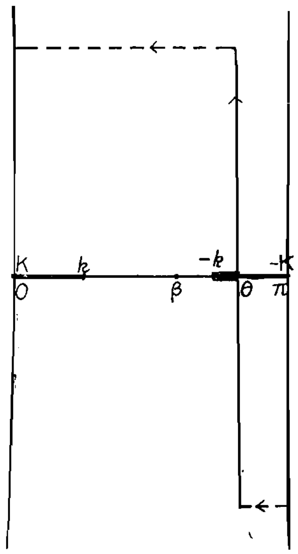


Fig. 5.

Next, we obtain the S part of the scattered pulse in a similar way for $\varphi>0$

$$u_i^s=\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{Q_i(\lambda,\kappa)}{k(\lambda-\kappa)}\,\frac{\sin\varepsilon b_1K}{K}\,e^{-ib_1Kt+i\left(\lambda x+\sqrt{K^2-\lambda^2}|y|+\frac{a_1k}{c}z\right)}dKd\lambda,$$

$$Q_1=\pm\frac{\frac{a_1^2k^2}{c^2}\left(\sqrt{k^2-\kappa^2}-B_2\right)(\lambda-\kappa)}{\pi i\sqrt{K-\kappa}\sqrt{K+\lambda}\left(\lambda^2+\frac{a_1^2k^2}{c^2}\right)}+\frac{\lambda\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}F^-(\lambda)H_2(\lambda)}{2\pi i\sqrt{K-\kappa}F^-(\kappa)\left(\lambda^2+\frac{a_1^2k^2}{c^2}\right)G(\lambda)}$$

$$Q_2=\mp\frac{1}{2\pi i}\sqrt{\frac{k-\lambda}{K-\kappa}}\,\frac{F^-(\lambda)H_2(\lambda)}{F^-(\kappa)G(\lambda)}$$

$$Q_3=\mp\frac{\frac{a_1k}{c}\lambda\{\sqrt{k^2-\kappa^2}-B_2\}(\lambda-\kappa)}{\pi i\sqrt{K-\kappa}\sqrt{K+\lambda}\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)}+\frac{\frac{a_1k}{c}\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}F^-(\lambda)H_2(\lambda)}{2\pi i\sqrt{K-\kappa}F^-(\kappa)\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)G(\lambda)}.\qquad(45)$$

Exchange the order of integration and transform the contour in λ plane to the path in δ' plane defined by $\lambda=K\cos\delta'=K\cos(\varphi+is)$ as shown in Fig. 5.

$$u_1{}^s=-\frac{\kappa\sqrt{k-\kappa}\sqrt{K+\kappa}H_2(\kappa)}{\left(\kappa^2+\frac{a_1{}^2k^2}{c^2}\right)G(\kappa)}D(t-\epsilon'x)-2Re\int_{s_1}^{s_2}\frac{iQ_1(\delta',\kappa)\sin(\varphi+is)}{k(\cos(\varphi+is)-\cos\beta)}ds$$

$$-\frac{1}{\pi}\int_{-s_2'}^{-s_1'}\frac{\lambda'\sqrt{k-\lambda'}\sqrt{K^2-\lambda'^2}F^-(\lambda')\bar{H}_2(\lambda')}{\sqrt{K-\kappa}F^-(\kappa)\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)k(\cos(\varphi+s)-\cos\beta)}ds$$

$$u_2{}^s=\pm\sqrt{\frac{k-\kappa}{K-\kappa}}\frac{H_2(\kappa)}{G(\kappa)}D(t-\epsilon'x)-2Re\int_{s_1}^{s_2}\frac{iQ_2(\delta',\kappa)\sin(\varphi+is)}{k\{\cos(\varphi+is)-\cos\beta\}}ds$$

$$\pm\frac{1}{\pi}\int_{-s_2'}^{-s_1'}\sqrt{\frac{k-\lambda'}{K-\kappa}}\frac{F^-(\lambda')\bar{H}_2(\lambda')}{F^-(\kappa)k\{\cos(\varphi+s)-\cos\beta)}ds$$

$$u_3{}^s=-\frac{\frac{a_1k}{c}\sqrt{k-\kappa}\sqrt{K+\kappa}H^2(\kappa)}{\left(\kappa^2+\frac{a_1{}^2k^2}{c^2}\right)G(\kappa)}D(t-e'x)-2Re\int_{s_1}^{s_2}\frac{iQ_3(\delta',\kappa)\sin(\varphi+is)}{k\{\cos(\varphi+is)-\cos\beta\}}ds$$

$$-\frac{1}{\pi}\int_{-s_2'}^{-s_1'}\frac{\frac{a_1k}{c}\sqrt{k-\lambda'}\sqrt{K^2-\lambda'^2}F^-(\lambda')\bar{H}_2(\lambda')}{\sqrt{K-\kappa}F^-(\kappa)\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)k\{\cos(\varphi+s)-\cos\beta\}}ds,\qquad(41)$$

$$\bar{H}_2(\lambda')=\frac{\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\kappa^2\right)\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda'^2\right)\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)\sqrt{(-k-\kappa')(K-\lambda')}}{\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda'^2\right)^4+\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)^2(\lambda'^2-k^2)(K^2-\lambda'^2)}$$

$$\frac{\pm\sqrt{(k+\kappa)(K-\kappa)}\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda^2\right)\left\{\lambda'\kappa+\frac{a_1{}^2k^2}{c^2}+\frac{B_1\frac{a_1k}{c}(\lambda'-\kappa)}{\sqrt{k^2-\kappa^2}}\right\}}{}$$

$$\frac{\times\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)\sqrt{(\lambda'^2-k^2)(K^2-\lambda'^2)}}{},\qquad(47)$$

$$\lambda'=K\cos(\varphi+s),$$

$$s_1'=\cos^{-1}\left(\frac{b_1(t_1-\varepsilon)}{r\sin\theta}\right)\qquad s_2'=\cos^{-1}\left(\frac{b_1(t_1-\varepsilon)}{r\sin\theta}\right),$$

$$e' = \cos \alpha'' e_x + \cos \beta'' e_y + \cos \gamma'' e_z,$$

$$\cos \alpha'' = \cos \beta \sin \gamma, \qquad \cos \beta'' = \sin \beta \sin \gamma, \qquad \cos \gamma'' = \cos \gamma,$$

where the first term vanishes for $\varphi > -\beta$.

In a similar way as above, we can obtain the displacements due to the incidence of the plane S pulse of the rectangular form.

Assume that the incident pulse has the form

$$\left.\begin{aligned} u_1{}^0 &= \cos \beta'' D\left\{t - \frac{1}{b}(\cos \alpha'' \cdot x \cos \beta'' \cdot y + \cos \gamma'' \cdot z)\right\} \\ u_2{}^0 &= -\cos\alpha'' D\left\{t - \frac{1}{b}(\cos \alpha'' \cdot x + \cos \beta'' \cdot y + \cos \gamma'' \cdot z)\right\} \\ u_3{}^0 &= 0 \end{aligned}\right\} \tag{48}$$

The scattered pulse for the incidence of the pulse of form (48) can be obtained by the inverse integral transform of the solution (40). The P part is for $\varphi > 0$.

$$\left.\begin{aligned} u^p &= e \frac{\sqrt{K-\kappa}}{\sqrt{k-\kappa}} \frac{K_1(\alpha)}{G(\alpha)} D(t - e \cdot x) \\ &\quad + 2\operatorname{grad} Re \int_{s_1}^{s_2} \frac{P'(\delta, \alpha) \sin (\varphi + is)}{K(\cos (\varphi + is) - \cos \alpha)} ds, \\ P'(\lambda, \kappa) &= \frac{1}{2\pi i} \sqrt{\frac{K-\lambda}{k-\kappa}} \frac{F^-(\lambda) K_1(\lambda, \kappa)}{F^-(\kappa) G(\lambda)}, \end{aligned}\right\} \tag{49}$$

where the first term expresses the reflected pulse and vanishes for $\varphi > -\alpha$, the second expresses the diffracted pulse.

Transform in the same way as the case of the S part for the incidence of the P pulse, then the S part is for $\varphi > 0$

$$\left.\begin{aligned} u_1{}^s &= -\frac{\kappa^2 \sqrt{K^2-\kappa^2} K_2(\kappa)}{\left(\kappa^2 + \frac{a_1{}^2 k^2}{c^2}\right) G(\kappa)} D(t - e' x) - 2Re \int_{s_1}^{s_2} \frac{iQ'_1(\delta', \kappa) \sin (\varphi + is)}{K(\cos (\varphi + is) - \cos \beta)} ds \\ &\quad - \frac{1}{\pi} \int_{-s_2'}^{-s_1'} \frac{\kappa \lambda' \sqrt{k-\lambda'} \sqrt{K^2 - \lambda'^2} F^-(\lambda') \bar{K}_2(\lambda')}{\sqrt{k-\kappa} F^-(\kappa) \left(\lambda'^2 + \frac{a_1{}^2 k^2}{c^2}\right) K(\cos (\varphi + is) - \cos \beta)} ds \\ u_2{}^s &= \pm \frac{\kappa K_2(\kappa)}{G(\kappa)} D(t - e' x) \pm 2Re \int_{s_1}^{s_2} \frac{iQ'_2(\delta', \kappa) \sin (\varphi + is)}{K(\cos (\varphi + is) - \cos \beta)} ds \\ &\quad \pm \frac{\kappa}{\pi} \int_{-s_2'}^{-s_1'} \sqrt{\frac{k-\lambda'}{k-\kappa}} \frac{F^-(\lambda') \bar{K}_2(\lambda')}{F(\kappa) K(\cos (\varphi + is) - \cos \beta)} ds \end{aligned}\right\}$$

$$u_3{}^s = -\frac{\frac{a_1 k}{c}\kappa\sqrt{K^2-\kappa^2}K_2(\kappa)}{\left(\kappa^2+\frac{a_1{}^2k^2}{c^2}\right)G(\kappa)}D(t-e'x) - 2Re\int_{s_1}^{s_2}\frac{iQ'_3(\delta',\kappa)\sin(\varphi+is)}{K(\cos(\varphi+is)-\cos\beta)}ds$$

$$-\frac{1}{\pi}\int_{-s_2'}^{-s_1'}\frac{\frac{a_1k}{c}\kappa\sqrt{k-\lambda'}\sqrt{K^2-\lambda'^2}F^-(\lambda')\bar{K}_2(\lambda')}{\sqrt{k-\kappa}F^-(\kappa)\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)K(\cos(\varphi+is)-\cos\beta)}ds, \tag{50}$$

$$Q_1' = \pm\frac{\frac{a_1{}^2k^2}{c^2}\left\{\frac{K^2}{2}-\kappa^2+\frac{\kappa\lambda}{2}-B^2(\lambda-\kappa)\right\}}{\pi i\sqrt{K-\kappa}\sqrt{K+\lambda}\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)}+\frac{\kappa\lambda\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}F^-(\lambda)K_2(\lambda)}{2\pi i\sqrt{k-\kappa}F^-(\kappa)\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)G(\lambda)}$$

$$Q_2' = \mp\frac{\kappa}{2\pi i}\sqrt{\frac{k-\lambda}{k-\kappa}}\frac{F^-(\lambda)K_2(\lambda)}{F^-(\kappa)G(\lambda)}$$

$$Q_3' = \mp\frac{\frac{a_1k}{c}\lambda\left\{\frac{K^2}{2}-\kappa^2+\frac{\kappa\lambda}{2}-B_2(\lambda-\kappa)\right\}}{\pi i\sqrt{K-\kappa}\sqrt{K+\lambda}\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)}+\frac{\frac{a_1k}{c}\kappa\sqrt{k-\lambda}\sqrt{K^2-\lambda^2}F^-(\lambda)K_2(\lambda)}{2\pi i\sqrt{k-\kappa}F^-(\kappa)\left(\lambda^2+\frac{a_1{}^2k^2}{c^2}\right)G(\lambda)}$$

$$\bar{K}_2(\lambda') = \sqrt{(k-\kappa)(K-\kappa)}\sqrt{(-k-\lambda')(K-\lambda')}\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda'^2\right)^2$$

$$\mp\left(\frac{K^2}{2}-\frac{a_1{}^2k^2}{2c^2}-\lambda'^2\right)\left(\lambda'^2+\frac{a_1{}^2k^2}{c^2}\right)\sqrt{(\lambda'^2-k^2)(K^2-\lambda'^2)}$$

$$\times\left\{\left(\frac{K^2}{2}-\kappa^2\right)\frac{\lambda'}{\kappa}-\frac{a_1{}^2k^2}{2c^2}-\frac{B_1a_1k(\lambda'-\kappa)}{\kappa c}\right\}, \tag{51}$$

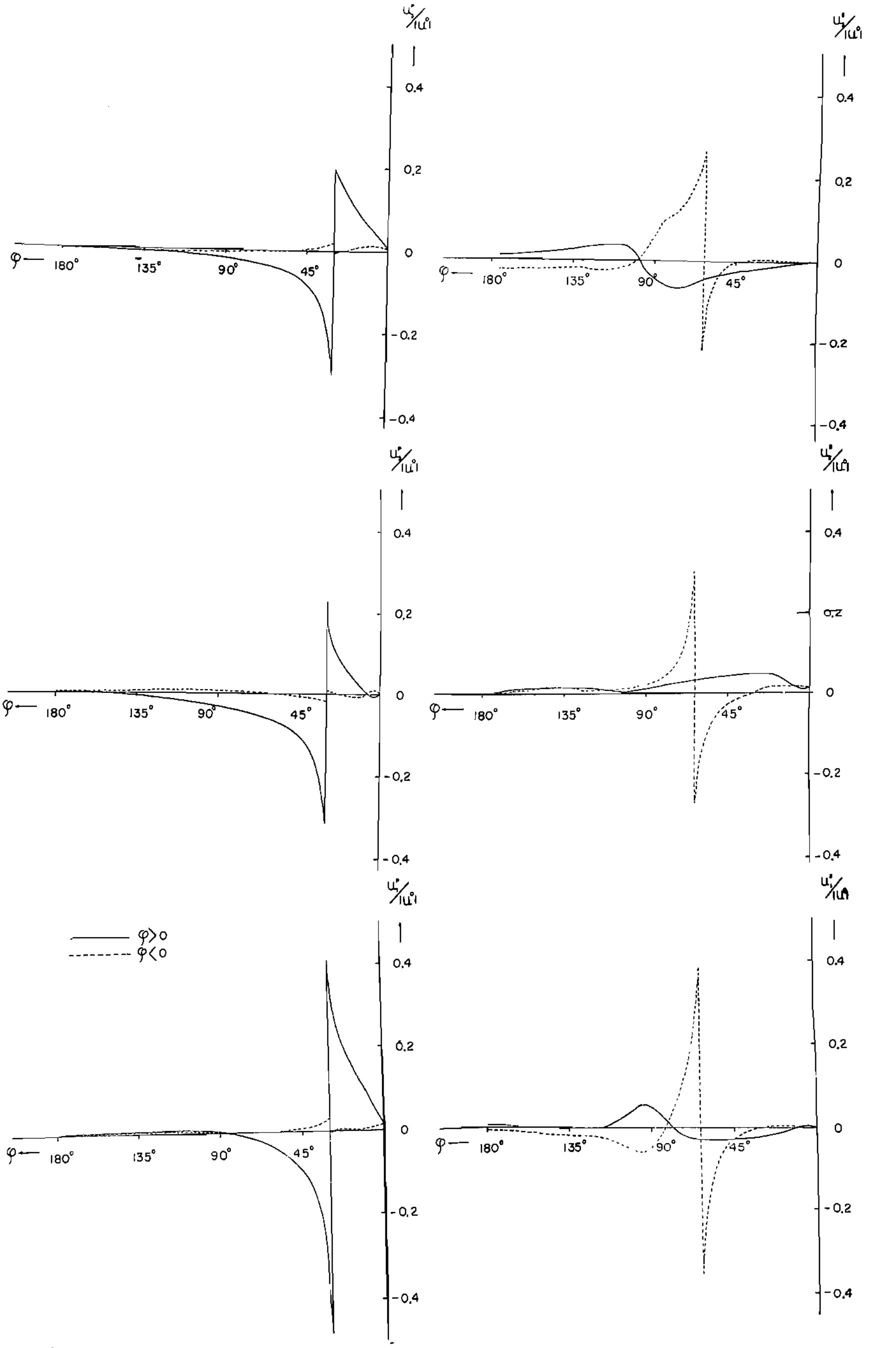
where for $|\varphi|<\beta$ the first term vanishes.

4. Numerical results

In our numerical examples, we assumed

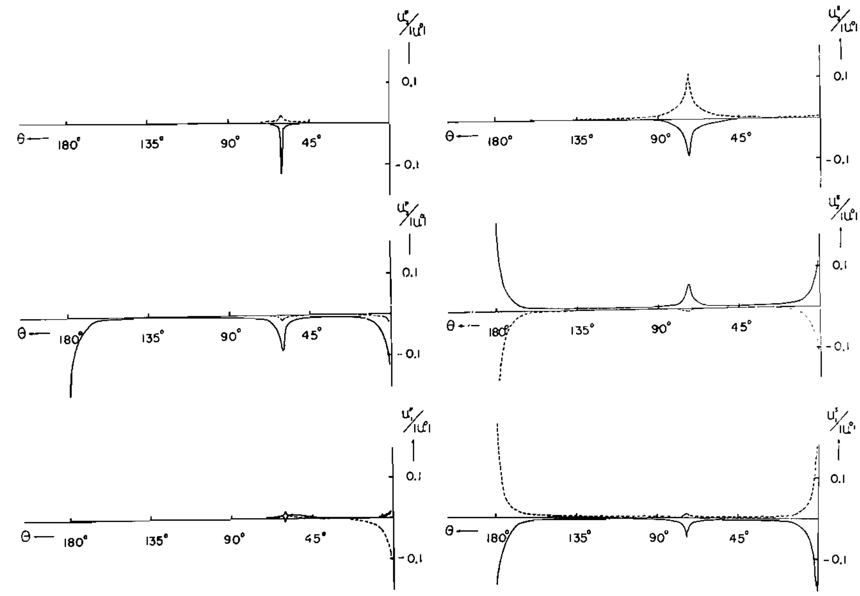
$$K=\sqrt{11/3}\,k$$
$$\alpha=30^\circ$$
$$\gamma=60^\circ$$

We investigated the diffraction picture resulting from the incidence of the plane longitudinal pulse, where the front of the incident pulse is always a plane intersecting the z-axis at the point $z=\frac{t}{c}$. This point is a vertex of the cone occupied by the diffracted pulse. We calculated the azimuthal distribution of the displacement of the diffracted pulse, which is the second and third term in the equation (44), at $\sqrt{\frac{a\varepsilon}{\gamma}}=0.1$ and $t=\frac{\gamma}{a}+\varepsilon$ for the diffracted



a) diffracted P pulse for $\theta=60°$ b) diffracted S pulse for $\theta=73°/4'$

Fig. 6. The φ-dependence of the amplitudes of displacement of the diffracted pulse for the incidence of P pulse.



a) diffracted P pulse for $\varphi=30°$ b) diffracted S pulse for $\varphi=63°6'39''$

Fig. 7. The θ-dependence of the amplacement of the diffracted pulse for the incidence of P pulse.

P pulse or $t=\frac{r}{b}+\varepsilon$ for the diffracted S pulse.

The φ-dependence of the displacements, u_1, u_2, u_3 of the diffracted pulse for θ=constant is rather similar to that of the amplitude in the case of the incidence perpendicular to the edge of the plane P pulse as shown in Fig. 6. That is, the amplitude of the diffracted P pulse increases at a uniform rate with the approach of φ to $\pm 30°$ and the phase is reverse at the shadow boundary of the incident P pulse and the reflected P pulse ; the displacements are a kind of double jerk. However, the composite displacement of this pulse with the reflected P pulse for $\varphi<0$ or the incident pulse for $\varphi>0$ is continuous at the boundary. While with respect to the diffracted S pulse the displacement varies continuously and becomes zero at $\varphi=63°6'39''$ for $\varphi>0$, the phase is reversed at the boundary $\varphi=-63°6'39''$ for $\varphi<0$ and the composite displacement of this pulse with the reflected S pulse is continuous at this angle. Being different from the two dimensional problem, u_1^s, u_3^s components of the displacement of the diffracted S pulse take large values near $\varphi=\pm\frac{\pi}{2}$, respectively. These correspond to the first term of the equation Q_t.

θ-dependence of the displacements of the diffracted pulse for φ=const. is shown in Fig. 7. They take maximum value at $\theta=60°$ or $73°14'$ and their phases are not reversed at this angle, which is different from their dependence on φ.

The writer wishes to express his hearty thanks to Prof. K. Sassa for his valuable suggestions.

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